Attractive interaction between pulses in a model for convection in binary mixtures

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Recent experiments on convection in binary mixtures have shown that the interaction between localized waves (pulses) can be repulsive as well as *attractive* and depends strongly on the relative *orientation* of the pulses. It is demonstrated that the concentration mode, which is characteristic of the extended Ginzburg-Landau equations introduced recently, allows a natural understanding of that result. Within the standard complex Ginzburg-Landau equation this would not be possible.

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Convection in binary mixtures exhibits extremely rich dynamics. Among the most striking features are localized wave packets which form stable, particlelike entities drifting through the system. They have been studied experimentally in great detail resulting in a large body of data [1–8]. On the theoretical side these results have raised a number of questions regarding their shape, regime of existence, stability, and drift velocity, which have been addressed by a number of authors employing full numerical simulations of the Navier-Stokes equations [9,10] as well as analytical and numerical studies of the complex Ginzburg-Landau equation (CGL) [11–14]. The simulations of the Navier-Stokes equations were able to reproduce various features of the experimental pulses and gave detailed insight into the concentration field which causes the oscillatory dynamics of the convection. The studies of the CGL showed that dispersion can provide a mechanism for the observed localization of the waves.

It has been pointed out, however, that various important qualitative features of the pulses cannot be captured within the CGL. Among them is the anomalously slow drift of the pulses and their stability behavior. In previous work I suggested that this is due to the relevance of a concentration mode which becomes an independent dynamical variable for the small mass diffusion in the liquids employed [15]. Based on this observation I derived a set of extended Ginzburg-Landau equations (ECGL's) which are characterized by an additional field, a slow concentration mode. Various analytical and numerical analyses showed that this extension is able to account for the difficulties discussed above regarding the dynamics of *single* pulses [15–19]. In the present communication I discuss an additional difficulty which arises in the description of the interaction *between* pulses.

An interesting set of experiments deals with the interaction of pulses in head-on and tail-on collisions [6]. It has been found that both types of collisions can result in stable bound pairs of pulses. In these experiments use is made of the fact that the drift velocity depends on the Rayleigh number and can in fact become opposite to the phase velocity ("backward" drifting pulse). To discuss the experimental results it is useful to call the "head" of a pulse that side towards which the waves travel inside the pulse. There are then two types of pairs consisting of counterpropagating pulses: those touching with their heads ("HH") and those touching with their tails ("TT"). In the experiments both types were found to be stable over a range of parameters. The HH pairs are only stable if the drift velocity of the pulses pushes them together. Thus the pulses in such a pair experience only a repulsive interaction. In the TT pairs the pulses exhibit a repulsive interaction at small distances; at larger distances, however, an *attractive interaction* can clearly be identified (see Figs. 16 and 17 in [6]) which may bind the pulses together even if their drift velocity tends to pull them apart. In the experimentally investigated regime this interaction was found to be very weak.

Within the conventional complex Ginzburg-Landau equations the pulse interaction arises from a complex cubic cross-coupling term (g, see below). Its real part induces a renormalization of the growth rate of one wave by the other. Its imaginary part affects the frequency (cross-phase modulation). In the regime of interest, the real part is negative, i.e., counterpropagating waves suppress each other. For pulses this leads naturally to a repulsive interaction. The imaginary part also contributes to the interaction. In the limit of vanishing dissipation, i.e., for coupled nonlinear Schrödinger equations, it has been shown that the cross-phase modulation can lead to stably bound pulse pairs if it has the correct sign [20,21]. In a two-dimensional analysis of the Navier-Stokes equations the opposite sign is, however, found for the regime in question [22]. In fact, this is also true for the coefficient c_i (see below) of the self-phase modulation. Soliton perturbation theory appears therefore not to be adequate in this regime. Even if a full, three-dimensional analysis should reveal the correct sign, the CGL would still not be able to account for the qualitatively different behavior of HH and TT pairs, since the phase velocity, which defines the head and the tail of the pulses, does not enter the CGL.

In the present communication I study the extended Ginzburg-Landau equations introduced earlier [15] and focus on the interaction between pulse pairs. Using numerical simulations I show that the additional concentration mode characteristic of these equations can lead to an interaction which depends on the orientation of the pulses as observed in the experiments [6]: the enhanced local growth rate of the convective mode behind a pulse can act as a bond within a TT pair. For HH pairs, however, the same mode leads to an increase in the repulsive interaction.

As discussed previously the slow mass diffusion in liquids necessitates the introduction of an additional (concentration) mode even for quite small convective amplitudes [15,16]. A minimal model describing the effect of the concentration mode on the buoyancy of the liquid [9,10] is given by [15]

$$\partial_t A + s\partial_x A = d\partial_x^2 A + (a + fC)A + c|A|^2 A + p|A|^4 A + g|B|^2 A + \cdots , \quad (1)$$

$$\partial_t B - s\partial_x B = d^*\partial_x^2 B + (a^* + f^* C)B + c^*|B|^2 B + p^*|B|^4 B + g^*|A|^2 B + \cdots , \quad (2)$$

$$\partial_t C = \delta\partial_x^2 C - \alpha C + h_2\partial_x(|A|^2 - |B|^2) + \cdots . \quad (3)$$

The complex amplitude of right- and left-traveling waves is given by A and B, respectively. The additional, real concentration mode C satisfies a diffusion equation with damping and is advected by the waves. In general all coefficients in (1) and (2) except for the group velocity s are complex. Here I will, however, consider the simplified case in which C affects only the growth rate of A and B, i.e., f is assumed real as well. The general case has been studied for short pulses in [18]. The localization of long pulses by the concentration mode has been discussed in detail in [17] by investigating the interaction between fronts (rather than pulses as is done in the present communication). The crossover between long and short pulses as well as their coexistence is addressed in [19].

Figure 1 shows a typical situation of two widely separated, essentially independent pulses. Their tails show the characteristic positive C field which enhances the growth rate of the respective convective amplitudes ($f > 0$). As discussed in detail previously [17,18] this can slow down the pulse. It is reasonable to expect that for sufficiently small distances between the pulses each pulse may feel not only the contribution to C from itself but also from the other pulse. This would lead to a further slow down and could amount to an attractive interaction.

To investigate the possibility of such an attractive interaction Eqs. (1)–(3) are solved numerically with periodic boundary conditions. Two pulses are placed in the system and the group velocity is chosen such that they collide with each other in a TT configuration, i.e., s is chosen negative at first. Once they have reached a steady state the equilibrium distance is measured as a function of the group velocity. The results are shown in Fig. 2 for $d = 0.15 + i$, $a = -0.24$, $f = 0.4$, $c = 2.4 + 2i$, $g = -10$, $p = -1.65 + 2i$, $\alpha = 0.02$, and $h_2 = 0.5$. The solid lines give the distance D between the pulses for two values of the "diffusion" coefficient δ of the concentration mode. Note that to leading order δ is due to differential buoyancy and advection; at that order the diffusion of the concentration field enters only the coefficient α [16]. The vertical arrows indicate the largest value s_{max} for which a stable bound pair is still obtained. If s is increased beyond s_{max} the pulses separate from each other and D diverges. The crucial test for the existence of an attractive interaction is a comparison of s_{max} with the

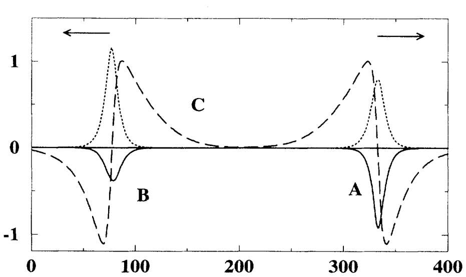

FIG. 1. Typical pair of widely separated pulses traveling to the left (B) and to the right (A), respectively. The solid and the dotted curves give the real and imaginary parts of the convective amplitudes of A and of B, respectively. Note that the two pulses happen to have different phases; their magnitudes are the same. The dashed line gives the concentration mode C. The parameters are given by $d = 0.15+i$, $a = -0.24$, $f = 0.4$, $c = 2.4 + 2i$, $g = -10$, $p = -1.65 + 2i$, $\alpha = 0.02$, $\delta = 0.25$, and $h_2 = 0.5$.

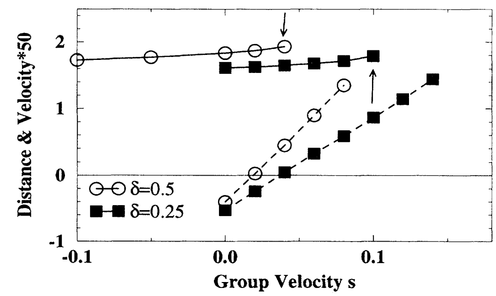

FIG. 2. Dependence of the distance D between the pulses (solid lines) and of the drift velocity v of a single pulse (dashed lines) on the group velocity s. Parameters are as in Fig. 1.

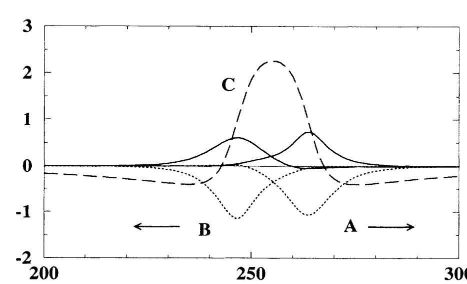

FIG. 3. Typical pair of stably bound pulses. The local growth rate is greatly enhanced between the pulses due to the accumulation of the concentration mode C (dashed line). This effectively binds the pulses together. The solid and the dotted curves give the real and imaginary parts of the convective amplitudes A and B, respectively. Parameters are as in Fig. 1.

value s_0 for which the drift velocity v of a single pulse becomes positive, i.e., the value s_0 for which the pulses would separate in the absence of any interaction. The numerically determined velocity of single pulses is given by the dashed lines. Clearly, it becomes positive well before s_{max}. Thus there is a finite range of parameters between s_0 and s_{max} in which the concentration mode acts as a bond which pulls the pulses together.

A typical bound pulse pair is shown in Fig. 3. It is characterized by a C field which is strongly enhanced between the pulses as compared to the widely separated case shown in Fig. 1 (note that the scales are different in Fig. 1 and Fig. 3). This does not occur for HH pairs. To the contrary, there the concentration mode enhances the repulsive interaction which is already present due to the suppressing cross-coupling term proportional to g in (1) and (2). Thus, within the ECGL a clear distinction exists between HH pairs and pairs of the TT type.

In conclusion, I have demonstrated that the attractive interaction between certain pulses observed experimentally [6] can be explained naturally if the concentration mode is taken into account. The latter also accounts for the observed dependence of the interaction on the relative orientation of the pulses. The standard complex Ginzburg-Landau equation does not capture these phenomena.

It would be interesting to study also the dynamics of collisions of pulses within the extended Ginzburg-Landau equations and compare them with results for the standard CGL [23]. In the experiments a drastic collapse of the TT pair is reported when the pulses are pushed together too strongly [6].

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[1] P. Kolodner, D. Bensimon, and C. Surko, Phys. Rev. Lett. **60**, 1723 (1988).
[2] D. Bensimon, P. Kolodner, and C. Surko, J. Fluid Mech. **217**, 441 (1990).
[3] J. Niemela, G. Ahlers, and D. Cannell, Phys. Rev. Lett. **64**, 1365 (1990).
[4] E. Moses, J. Fineberg, and V. Steinberg, Phys. Rev. A **35**, 2757 (1987).
[5] P. Kolodner, Phys. Rev. Lett. **66**, 1165 (1991).
[6] P. Kolodner, Phys. Rev. A **44**, 6466 (1991).
[7] P. Kolodner, Phys. Rev. E **48**, 4187 (1993).
[8] P. Kolodner, Phys. Rev. E **50**, 2731 (1994).
[9] W. Barten, M. Lücke, and M. Kamps, Phys. Rev. Lett. **66**, 2621 (1991).
[10] W. Barten, M. Lücke, M. Kamps, and R. Schmitz, Phys. Rev. E **51**, 5662 (1995).
[11] O. Thual and S. Fauve, J. Phys. (Paris) **49**, 1829 (1988).
[12] L. Pismen, Phys. Rev. A **35**, 1873 (1987).
[13] B. Malomed and A. Nepomnyashchy, Phys. Rev. A **42**, 6009 (1990).
[14] V. Hakim and Y. Pomeau, Eur. J. Mech. B Suppl. **10**, 137 (1991).
[15] H. Riecke, Phys. Rev. Lett. **68**, 301 (1992).
[16] H. Riecke, Physica D **61**, 253 (1992).
[17] H. Herrero and H. Riecke, Physica D **85**, 79 (1995).
[18] H. Riecke, Physica D (to be published).
[19] H. Riecke and W.-J. Rappel (unpublished).
[20] B. Malomed, Phys. Rev. A **45**, 8321 (1992).
[21] T. Ueda and W. Kath, Phys. Rev. A **42**, 563 (1990).
[22] W. Schöpf and W. Zimmermann, Phys. Rev. E **47**, 1739 (1993).
[23] H. Brand and R. Deissler, Phys. Rev. Lett. **63**, 2801 (1989).